Financial Markets and the Real Economy: a statistical field perspective on capital allocation and accumulation

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Purpose of the paper

- We have previously developed and applied a field formalism, derived from statistical physics, to economic models with large number of agents

- Compared to standard approaches (representative agent, ABM, mean field):
  - No simulations needed
  - collective states are derived directly from the form of interactions between agents rewritten in terms of fields
  - Possibility to study impact of collective states on individual agents

Purpose of the paper

This paper:

- Provides a step-by-step method to directly translate a classical economic framework with a large number of agents into a field-formalism model

- Applies this methodology to model the interactions between a large number of:
  - Investors, seen as financial market
  - Producers, seen as the real economy

- Studies capital allocation and accumulation resulting from these interactions in:
  - A static environment
  - A dynamic environment

Field formalism for a large number of agents

- Method developed previously by the authors
- Translate a dynamic system with a large number of agents into a statistical field model
- The N agents dynamics is described by:
  - a "field": an abstract function that encodes the agents
  - a function of this field: the "action" of the system that encompasses the dynamic system

Field formalism for a large number of agents

A system of dynamic equations:

$$\frac{d\mathbf{A}_i(t)}{dt} - \sum_{j,k,l...} f\left(\mathbf{A}_i(t), \mathbf{A}_j(t), \mathbf{A}_k(t), \hat{\mathbf{A}}_l(t), \hat{\mathbf{A}}_m(t) ...\right)$$

Plus eventually some optimization problems (utility…)

TRANSLATION

Are replaced by a field ψ(A), and a field action S(ψ):

- The field ψ(A) keeps tracks of the set of agents:

  The dynamic variable A becomes the argument of the function

  One field for each type of agent
- The field action S(ψ) encodes agents' dynamics and interactions

  Its form depends directly on the dynamic equations

Field formalism for a large number of agents

- The solutions to the minimization equations of the action functional S(ψ) are called the ***background fields*** of the system

- They describe the potential equilibria of the system. They:
  - Characterize the collective states of the system
  - May be used to compute average quantities of the system
  - Structure the interactions between agents
  - Condition individual dynamics

- Once the background field found, expanding the action functional S(ψ) around this background yields the individual dynamics of generic agents in a given background field

Field formalism Advantages

The field formalism allows to:

- Deal analytically with the full system

- Study the emergence of some particular states of the system, the "background fields", in which individual agents evolve

- Describe the collective behaviors of the system for a large number of agents

- Keep track of individual dynamics and describe generic emerging agents

This field formalism also allows a mutual interpretation between micro and macro levels.

Application: microeconomic framework

- Two groups of agents: producers and investors
- Producers represent the real economy
- Investors represent the financial markets

Application: microeconomic framework

- Firms

  - Large number of firms in different sectors X
  - Compete by producing differentiated goods
  - Are endowed with physical capital K
  - Their physical capital depends on the capital lent by investors
  - May shift between sectors to improve their returns and attract investors
  - Reward their investors:
    - Pay dividends
    - Through the valuation of their stock prices

Application: microeconomic framework

- Investors:
  - hold financial capital $\hat{K}$
  - allocate it between firms across sectors according to:
    - Investment preferences
    - Expected returns of firms R(K,X)
    - Stock prices variations on financial markets
  - Move along sectors based on firms' expected long-run returns
  - Increase their capital through dividends and stock prices

Application: microeconomic framework

The dynamics follows the following pattern:

1. Investors allocate their capital between firms
2. Firms use capital to produce
3. Short-term returns are generated
4. Capital is returned to investors

1. Financial capital allocation

- Investors allocate their capital to producers:

$$\hat{K}_j^{(i)}(t) = \left(\frac{F_2\left(R\left(K_i, X_i\right)\right) G\left(X_i - \hat{X}_j\right)}{\sum_l F_2\left(R\left(K_l, X_l\right)\right) G\left(X_l - \hat{X}_j\right)} \hat{K}_j \right)(t)$$

Share of capital invested in firm *i*. Depends on expected returns

Capital invested in firm *i* by firm *j*

2. Firms' disposable capital

- Producers' capital is the sum of capital invested

$$K_i(t+\varepsilon) = \sum_j \hat{K}_j^{(i)} = \sum_j \frac{F_2\left(R\left(K_i(t), X_i(t)\right)\right) G\left(X_i(t) - \hat{X}_j\right)}{\sum_l F_2\left(R\left(K_l(t), X_l(t)\right)\right) G\left(X_l(t) - \hat{X}_j\right)} \hat{K}_j(t)$$

Capital invested in firm *i* by firm *j*

Capital at the beginning of the period

3. Short-term returns

- The short-term returns of a firm are composed of:
  - Dividend r_i that is both:
    - firm-dependent
    - Sector-X and capital-K dependent

$$r_i = r(K_i, X_i) - \gamma \sum_j \delta(X_i - X_j) \frac{K_j}{K_i}$$

  - Variations in stock prices

$$\frac{\dot{P}_i}{P_i} = F_1\left(\frac{R(K_i, X_i)}{\sum_l R(K_l, X_l)}\right)$$

- R is the expected long-run return of the firm

  It depends on K and X

4. Financial payoffs

$$\hat{K}_j(t+\varepsilon) - \hat{K}_j(t) = \sum_i \left(r_i + \frac{\dot{P}_i}{P_i}\right)\hat{K}_j^{(i)} = \sum_i \left(r_i + F_1\left(\frac{R(K_i, X_i)}{\sum_l R(K_l, X_l)}, \frac{\dot{K}_i(t)}{K_i(t)}\right)\right)\hat{K}_j^{(i)}$$

Return of firm *i*

Capital invested by investor *j* in firm *i*

Financial capital variation between two periods

Dynamics within sectors' space

- The model is closed by considering that both producers and investors move within the sectors' space towards higher returns (equations given in the text)

- Main characteristics:

  - Producers are:
    - Driven by the perspective of higher returns
    - Detered by competion in the targeted sector

  - Investors are driven by:
    - Perspective of higher returns relative to the neighbouring sectors
    - Stock prices variations

Framework : synthesis

- Two variables shape the landscape and condition the form of the state of the system:
  - Short-term returns:
    - Dividends r
    - Price variations

    Short-term returns depends on sectors, capital invested, competition…

  - Expected long-term returns R:
    - Impact stock prices: R depends on sectors X and capital invested K

- Variations of these quantities permanently modify the collective state of the system

  They induce a dynamics in potential equilibria

Field translation of the system

To inspect these points, we translate the system in terms of fields

- The field translation involves:
  - Two fields:
    - One for the real economy: $\Psi(K, X)$
    - One for the financial markets: $\hat{\Psi}\left(\hat{K}, \hat{X}\right)$
  - A field action functional S, from which we derive the collective state of the system

Field translation of the system

$$
\begin{aligned}
S = & -\int \Psi^{\dagger}(K,X)\left(\nabla_X\left(\frac{\sigma_X^2}{2}\nabla_X - \nabla_X R(K,X) H(K)\right) - \tau\left(\int |\Psi(K',X)|^2 dK'\right)\right. \\
& \left. + \nabla_K\left(\frac{\sigma_K^2}{2}\nabla_K + u\left(K,X,\Psi,\hat{\Psi}\right)\right)\right)\Psi(K,X)\, dK\, dX \\
& -\int \hat{\Psi}^{\dagger}\left(\hat{K},\hat{X}\right)\left(\nabla_{\hat{K}}\left(\frac{\sigma_{\hat{K}}^2}{2}\nabla_{\hat{K}} - \hat{K} f\left(\hat{X},\Psi,\hat{\Psi}\right)\right) + \nabla_{\hat{X}}\left(\frac{\sigma_{\hat{X}}^2}{2}\nabla_{\hat{X}} - g\left(K,X,\Psi,\hat{\Psi}\right)\right)\right)\hat{\Psi}\left(\hat{K},\hat{X}\right)
\end{aligned} \tag{42}
$$

Field describing producers

Field describing investors

Functions *u*,*f*,*g*,*H* encode the micro-framework (given in the text)

Resolution of the field model

- The paper computes the background fields (i.e. the collective states) for the real economy and the financial markets

- From a sector perspective, the collective states determine:
  - Capital average distribution per sector
  - Firms' concentration within sectors

These quantities depend on external parameters, such as:

- Changes in expected returns
- Changes in dividends (technological advances)
- Their dynamics, when these external conditions evolve

Results : static environment

- put differently, the collective configurations are characterized, at each point of the sectors' space, by:
  - The equilibrium average capital
  - The number of firms

Results : static environment

- Sectoral capital accumulation depends on the environment, i.e.:
  - Short-term returns (dividends and price variations)
  - Expected long-term returns
  - Relative expected long-term returns: sectoral accumulation is not local, depends on the landscape
  - There is a partial trade-off between these variables

- The number of firms per sector depends on:
  - the average level of capital invested in the sector
  - The expected long-term return

Results : static environment

For each sector, three possible patterns of accumulation (equilibrium values) emerge:

- Pattern I:
  - Dividends in short-term returns are determinant for accumulation
  - Sectors with few firms and low average capital
- Pattern II:
  - Sectors' short and long-term returns drive capital accumulation
  - Sectors with intermediate-to-high capital firms
- Pattern III:
  - Higher expectations of long-term returns drive massive inputs of capital
  - Instability in capital accumulation may arise among and within sectors: thresholds effects appear in average capital

Results : static environment

- The equilibrium may be unstable:
  - Changes in parameters or expectations may induce changes in portfolio allocation between sectors.
  - May leave some sectors deserted

- At a macro-timescale:
  - Any deviation from an equilibrium drives a sector towards the next stable equilibrium, zero included
  - When there is none, towards infinity

- This instability is relative and context-dependent:
  - Variations of parameters in some sectors may propagate to other sectors

Results : dynamic environment

To account for this systemic instability, we adopt a wider approach to our model:

We consider a dynamic system involving:

- Average capital per sector
- Endogenized long-term expected returns (most volatile parameter)

This dynamic system differs from those in standard economic:

- In economics the dynamics is usually studied around a static equilibria
- We consider the dynamic interactions between potential equilibria and expected long-term returns

Results : dynamic environment

- Average capital per sector interact with:
  - One another
  - Long-term expected returns

- Some solutions of this dynamic system are oscillatory:

  Changes in one or several sectors may propagate over the whole sectors' space

- Pattern III sectors (high capital, high expected return):
  - Favored by fluctuations when expectations are highly sensitive to capital variations
  - These sectors drive capital from neighboring sectors

Conclusion

- This example shows that field formalism allows detailed analysis of systems with large number of agents

- Next paper: step-by-step method to compute agents dynamics within background states

Thank you for your attention